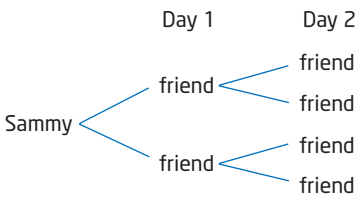
3.2 Work With Exponents

Suppose the trend in the cartoon continues: every day each new customer tells two new friends at school about the Barney Burger. How many new customers will Barney get each day?



Investigate

How can you use exponential models to describe growth patterns?

1. Copy and complete the table. In the last column, write the number of new customers as a power of 2.

New Customers After Sammy Tells His Two Friends

Day	New Customers	Expanded Form	Power
1	2	2	2^1
2	4	2×2	2^2
3			
4			

2. Barney's is open 7 days a week. Use this model to determine how many new customers Barney should expect on Day 7. Explain how you found your answer.

3. Use this model to determine how many new customers Barney should expect on Day 14. Is this answer realistic? Why or why not?

4. Estimate the number of students at your school. How long would it take for everyone at your school to find out about Barney's? Describe how you found your answer, and identify any assumptions you made.

5. Suppose that each new customer told three friends about Barney's, instead of two, and that this trend continued.

Use exponents to help explain your answers to the following.

a) How many new customers should Barney expect after 2 days?

b) How many new customers should Barney expect after 4 days?

c) How much more quickly would word reach all the students at your school? Explain.

6. **Reflect** Explain how exponents are useful in describing growth patterns.

A **power** is a product of identical factors and consists of two parts: a **base** and an **exponent**.

2^4 is a power

base: 2 — exponent: 4

The base is the identical factor, and the exponent tells how many factors there are.

$2^4 = 2 \times 2 \times 2 \times 2$

exponential form: 2^4 — expanded form: $2 \times 2 \times 2 \times 2$

Powers are useful for expressing repeated multiplication.

Example 1 Evaluate Powers

Write in expanded form, and then evaluate.

a) 2^5 **b)** $(-3)^3$ **c)** $(-3)^4$

d) -3^4 **e)** 3.5^3 **f)** $\left(\frac{2}{3}\right)^3$

Solution

a) $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

b) $(-3)^3 = (-3) \times (-3) \times (-3)$
$= -27$

There is an odd number of negative factors. The answer is negative.

c) $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$
$= 81$

There is an even number of negative factors. The answer is positive.

d) $-3^4 = -(3 \times 3 \times 3 \times 3)$
$= -81$

The base of this power is 3, not -3. The negative sign in front makes the result negative.

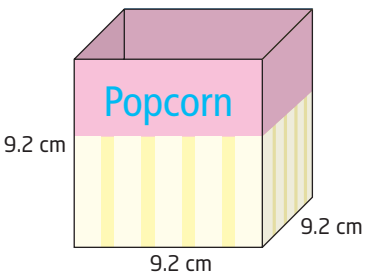
e) $3.5^3 = 3.5 \times 3.5 \times 3.5$
$= 42.875$

C 3.5 y^x 3 =

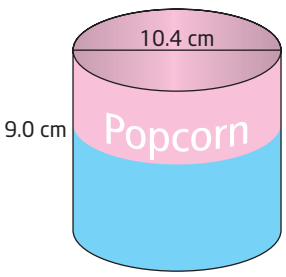
f) $\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$
$= \frac{2 \times 2 \times 2}{3 \times 3 \times 3}$
$= \frac{8}{27}$

To multiply fractions, I multiply numerators together and multiply denominators together.

Example 2 Apply Exponents to Solve Problems



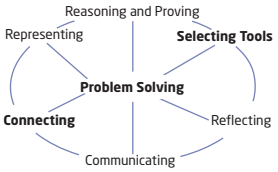
Mega-Box



Jumbo Drum

Which container holds more popcorn? How much more? Assume that each container is filled just to the top. Round your answer to the nearest cubic centimetre.

Solution



Mega-Box

The Mega-Box is in the shape of a cube. Apply the formula for the volume of a cube.

$V = s^3$ — *s* is the side length of the cube.
$= 9.2^3$ — C 9.2 y^x 3 =
$= 778.688$

The Mega-Box holds about 779 cm^3 of popcorn.

Jumbo Drum

The Jumbo Drum is in the shape of a cylinder. Apply the formula for the volume of a cylinder.

$V = \pi r^2 h$ — *r* is the radius of the base and *h* is the height of the cylinder.
$= \pi(5.2)^2(9.0)$ — The radius is half the diameter: 10.4 ÷ 2 = 5.2
$\doteq 764.54$ — C π × 5.2 x^2 × 9 =

The Jumbo Drum holds about 765 cm^3 of popcorn.

The Mega-Box holds 14 cm^3 more popcorn than the Jumbo Drum. — 779 − 765 = 14

Key Concepts

- Powers are a useful way to express repeated multiplication. For example,
  $4 \times 4 \times 4 = 4^3$
- A power consists of a base and an exponent, e.g., 4^3.
  - The base is the identical factor.
  - The exponent tells how many factors there are.
- Powers sometimes appear in formulas. When evaluating expressions involving powers, follow the correct order of operations.

Communicate Your Understanding

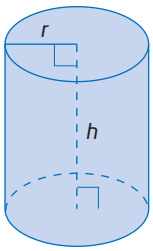
C1 Identify the base and the exponent of each power.

a) 3^4 **b)** $\left(\frac{1}{2}\right)^4$ **c)** $(-2)^6$ **d)** -2^6 **e)** 1.2^2

C2 **a)** Evaluate each power in question 1.

b) Explain why the answers to parts c) and d) are different.

C3 The first step in evaluating the volume of a cylinder is to substitute the known values for r and h into the formula for the volume of a cylinder: $V = \pi r^2 h$. Describe the next step.



C4 Which expressions would you evaluate using a calculator? Explain.

a) 2^3 **b)** $(-4)^2$ **c)** $(1.25)^4$

d) -8^2 **e)** 7^6 **f)** $(-0.1)^3$

Practise

For help with questions 1 to 5, see Example 1.

1. Which is $6 \times 6 \times 6 \times 6$ written as a power?

A 64 **B** 6^4

C 4^6 **D** 1296

2. Which is 3^5 written in expanded form?

A 3×5 **B** $5 \times 5 \times 5$

C $3 \times 3 \times 3 \times 3 \times 3$ **D** 243

3. Write each expression as a power.

a) $(-5) \times (-5) \times (-5)$

b) $1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05$

c) $\left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right)$

4. Write each power in expanded form. Then, evaluate the expression.

a) $(-4)^3$ **b)** 0.8^2 **c)** $\left(\frac{3}{4}\right)^4$

5. Evaluate.

a) 9^3 **b)** $(-7)^2$ **c)** -2^4

d) $\left(\frac{5}{6}\right)^3$ **e)** $\left(-\frac{2}{3}\right)^4$ **f)** 1.2^2

g) 1^8 **h)** $(-1)^{55}$ **i)** 0.5^3

6. Evaluate. Remember to use the correct order of operations.

a) $2^5 \div 4^2$ **b)** $5^3 - 5^2$ **c)** $1^3 + 1^6 - 1^2$

d) $(3^2 - 4^2) + (3^4 - 4^3)$ **e)** $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2$ **f)** $500(1.08)^5$

7. Substitute the given values into each expression. Then, evaluate the expression. Round your answers to one decimal place where necessary.

a) $6s^2$ $\quad s = 5$

b) πr^2 $\quad r = 2.5$

c) $a^2 + b^2$ $\quad a = 3, b = 4$

d) $\pi r^2 h$ $\quad r = 2.3, h = 5.2$

e) $\frac{4}{3}\pi r^3$ $\quad r = 1.5$

f) $x^2 - 2x - 24$ $\quad x = -6$

Technology Tip

If your calculator does not have a π key, use 3.14 as an approximate value for π.

Connect and Apply

8. a) Evaluate each power.

$(-2)^2 \quad (-2)^3 \quad (-2)^4 \quad (-2)^5$

b) Examine the signs of your answers. What pattern do you notice?

c) Explain how you can tell the sign of the answer when a power has a negative base. Create and use examples of your own to illustrate your explanation.

9. Listeria is a type of bacteria that can cause dangerous health problems. It doubles every hour. The initial population of a sample of Listeria is 800.

Time (min)	Population of Listeria
0	800
60	1600
120	
180	
240	

a) Copy and complete this table, which shows the population of Listeria over time.

b) Construct a graph of population versus time. Use a smooth curve to connect the points. Describe the shape of the graph.

c) What will the population be after
- 1 day?
- 2 days?

d) The symptoms of food poisoning can start as quickly as 4 h after eating contaminated food or as long as 24 h later. Discuss why some types of food poisoning begin quickly and others much more slowly.

Did You Know?

Food and water contaminated with E. coli (escherichia coli) can be very dangerous, but infections are easily treatable with certain antibiotics.

10. E. coli is a type of bacteria that lives in our intestines and is necessary for digestion. It doubles in population every 20 min. The initial population is 10.

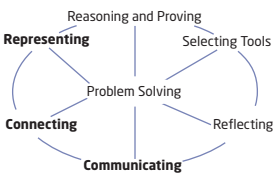
a) Copy and complete the table. Refer to your table from question 9 to complete the second column.

Time (min)	Population of Listeria	Population of E. Coli
0		
20		
40		
60		
80		
100		
120		

b) When will the population of E. coli overtake the population of Listeria?

c) What population will the two cultures have when they are equal?

11. The durations (lengths of time) of musical notes are related by powers of $\frac{1}{2}$, beginning with a whole note. Copy and complete the table.



Note	Symbol	Duration (in beats)	Power Form
whole	𝅝	1	
half	𝅗𝅥	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1$
quarter	♩	$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2$
eighth	♪		
sixteenth	𝅘𝅥𝅯		
thirty-second	𝅘𝅥𝅰		

12. Refer to question 11. Look at the pattern in the last column. Extend this pattern backward to write the power form for a whole note. Does this answer make sense? Use a calculator to evaluate this power. Describe what you observe.

13. Chapter Problem Alysia has selected the letter E to design the logo for her school team, the Eagles.

The design will be used to make different-sized crests for clothing such as jackets, sweaters, and baseball caps. The height of the crest is twice the width. How can Alysia make sure that, when the crest is made larger or smaller, the proportions will not change?

a) Find an expression for the area of the crest in terms of the width.

b) Determine the area of a crest with a width of 8 cm.

c) Determine the height of a crest with an area of 72 cm^2.

14. Uranium is a radioactive material that emits energy when it changes into another substance. Uranium comes in different forms, called isotopes. One isotope, U-235, has a half-life of 23 min, which means that it takes 23 min for a sample to decay to half its original amount.

a) Suppose you started with a 100-mg sample of U-235. Copy and complete the table.

Number of Half-Life Periods	Time (min)	Amount of U-235 Remaining (mg)	Expression
0	0	100	
1	23	50	$100\left(\frac{1}{2}\right)^1$
2	46		$100\left(\frac{1}{2}\right)^2$
3			
4			

Did You Know?

Uranium is used as a fuel source in nuclear fission reactors, which provide 48% of Ontario's electrical power.

CANDU (**Can**ada **D**euterium **U**ranium) reactors are among the safest nuclear power generators in the world.

b) Construct a graph of the amount, in milligrams, of U-235 remaining versus time, in minutes. Describe the shape of the graph.

c) Approximately how much U-235 will remain after 2 h?

d) How long will it take until only 1 mg of U-235 remains?

e) Use the pattern in the table to write an expression, using powers of $\frac{1}{2}$, for the original amount of U-235. Does this make sense?

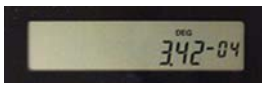
Literacy Connections

Scientific notation is a convenient way to write very large or very small numbers. In scientific notation, the value is expressed as the product of a number between 1 and 10 and a power of 10.
For example,
$56\,000\,000\,000 = 5.6 \times 10^{10}$
$0.000\,342 = 3.42 \times 10^{-4}$

Scientific calculators express numbers in scientific notation when there are too many digits for the display. Multiply 1234 by 1000. Repeat until the output appears in scientific notation.

You can enter a number in scientific notation into a scientific calculator.
For example, to enter 5.6×10^{10}, press
C 5.6 EE 10 =

To enter 3.42×10^{-4}, press
C 3.42 EE 4 ± =



Not all calculators show scientific notation, or let you enter such numbers, in exactly the same way. Experiment or refer to the user's manual for your calculator.

Extend

15. Uranium-233 is another isotope that is used in nuclear power generation. 1 kg of U-233 can provide about the same amount of electrical power as 3 000 000 kg of coal. This number can be written in **scientific notation** as 3×10^6.

a) Another isotope of uranium, U-238, has a half-life of 4 500 000 000 years. Write this number in scientific notation.

b) What is the half-life of U-238, in seconds? Write your answer in scientific notation.

c) The number 6.022×10^{23} is a very important number in chemistry. It is called "one mole." One mole is the amount of a substance that contains as many atoms, molecules, ions, or other elementary units as the number of atoms in 12 g of carbon-12. Carbon-12 is the basic building block of living things. Write one mole in standard notation.

d) Describe any advantages you see to using scientific notation.

16. Refer to the cartoon at the beginning of the section. Suppose that every new customer returns to Barney's every day for lunch, in addition to recruiting two new customers.

a) How many customers in total will Barney have
- 2 days after Sammy's first visit?
- 5 days after Sammy's first visit?

b) On which day will Barney's reach 500 new customers for lunch?

c) Write an expression that gives the total number of new lunch customers n days after Sammy's first visit.

d) Describe any assumptions you must make in finding your answers.

17. Math Contest Determine the last digit of the number 3^{1234} when written in expanded form. Justify your answer.

18. Math Contest If $3^x = 729$, the value of x is

A 3 **B** 5 **C** 6 **D** 7 **E** 8

19. Math Contest Numbers are called perfect powers if they can be written in the form x^y for positive integer values of x and y. Find all perfect powers less than 1000.

20. Math Contest x^x is always greater than y^y as long as $x > y$. For what whole-number values of x and y is $x^y > y^x$? Justify your answer.